Pre-service teachers' understandings of word problems

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Mathematics curricular statements refer to problem solving, in particular word problems, as a means of relating the curriculum to "reality". Research indicates that pre-service primary teachers have little knowledge of word problems despite the strong emphasis on their importance within the curriculum. This study reports on the understandings of word problems types of pre-service primary teachers. A number of activities were given as part of a mathematics education method course. Educational implications of these findings are discussed and recommendations proffered as to the amelioration this situation.

Introduction

The Australian Curriculum and Standards Framework: Mathematics (Board of Studies, 1995) details the learning outcomes for all students in the compulsory years of schooling in this key learning area. This framework advises teachers that by the completion of Level 3 (Year 4 students - about 10 year old children) students are expected to be able to "generate word problems using specified numbers and operations" (p. 54). The American Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989) (NCTM) similarly advises teachers of elementary classes that students should have multiple experiences in solving realistic word problems.

Numerous researchers (e.g., Stacey & Groves, 1978; Verschaffel, De Corte & Borghart, 1996; Woodward, 1991) have investigated the role of word problems in mathematics education method courses for pre-service teachers. Stacey & Groves (1987) introduced problem solving as part of their first year method class. Their students, working in collaborative groups, were introduced to a wide variety of problems including word problems. They found that many pre-service teachers were somewhat troubled by the lack of a "tight" course structure; others had difficulty working collaboratively; whilst only a few "thrived on the freedom allowed" (p. 340). Verschaffel et al. (1996) looked at the artificiality of many word problems and the inability of a number of pre-service teachers to use their knowledge of reality to solve a series of problems, half of which were problematic problems. Problematic problems require students to view the problems in context and to use their knowledge of real situations to obtain an accurate solution. They point out the necessity to change "the teachers' own conceptions and beliefs about the importance of real-work knowledge in arithmetic word problems solving" (p. 4-394). From a somewhat different perspective, Woodward (1991) asserted that teachers will not teach problem solving until they become proficient problem solvers themselves. Pre-service teachers experienced a variety of strategies (such as pattern identification, table building, graphic representations) as a means of solving a diverse range of problems. He found that pre-service teachers gained confidence in their ability to teach mathematics problem solving through this experience. Fennema, Carpenter, Levi, Franke, & Empson, (1997) in their Cognitively Guided Instruction course use a teacher's knowledge of word problem categories as a means of gaining a full understanding of their students' mathematical thinking.

If, as research has shown, teachers are required to have an understanding of word problems to effectively teach their students to become competent problem solvers, then they should have an understanding of the categories of word problems their students are likely to meet. This study was conducted in order to ascertain whether these pre-service teachers had gained an understanding of problem categories from their experiences in the methods course they had undertaken.

The Study

This study involved 42 pre-service primary school teachers taking a mathematics method as part of their Graduate Diploma of Education.

A 1-3-6 method of building a collaborative decision regarding the categorisation of word problems was employed. Because of space constraints, this methodology is explained in the activities below rather than as a separate section. The response sheet for each student was collected in their final group and analysed to determine their categorisation and generation of word problems.

The activities described below enable an analysis of individual and groups of pre-service teachers' ability to come to grips with word problems. The findings should provide an indication of additions may be needed to method courses to enable these pre-service teachers cope with the coming demands of the curriculum they will be required to teach.

1. Students were initially presented with 36 unclassified word problems and asked to classify them into twelve different categories. They were told that there were six problems involving either addition or subtraction and six involving either multiplication or division. Students were asked to work individually and write down the number of the problems which they considered belonged in the same category. Although no category headings were provided to the students, these are shown in Table 1.

Table 1. Categories and example of word problem

Type of word problem	Category	Example of word problem
Additive	Joining	*Mary has F5¢ in her pocket. On the way home she found 10¢ in the grass. How much money does she have now?*
	Separating	*Richard had 15 marbles. He lost 7 of them in a game. How many marbles has he left?*
	Equalising-add on	*Jessica has 15¢. Alexander has 35¢. How much more does Jessica need to save to have the same amount as Alexander?*
	Equalising-take away	*Irene has 25 marbles and Peter has 17 marbles. How many does Irene need to lose to have the same number as Peter?*
	Part-part-whole	*In the purse there is a 10¢ coin and a 20¢ coin. How much is in the purse?*
	Comparison	*In his bag Igor had 25 small marbles and 16 large marbles. How many more small marbles are there than large marbles?*
Multiplicative	Comparison multiplication	*Margaret has written 12 lines of her story. James has written three times as many lines as Margaret. How many lines has James written?*
	Comparison division	*Jillian has already written 46 lines of her story so far. This is twice as many lines as David has written. How many lines has David written?*
	Cartesian product	*Elizabeth has 3 summer skirts, 4 blouses and 2 pair of shoes. How many different outfits can she wear?*
	Partition division	*Ms Vormann bought 15 chocolate bars to give to the children at the party. There were 5 children at the party. If each child received the same number of chocolate bars, how many chocolate bars did each child receive?*
	Quotition division	*Mr. Tsitas bought 15 chocolate bars to give to the children at the party. Each child was given 3 chocolate bars. How many children were at the party?*
	Equal groups	*Each week Ms Bye bought 5 pieces of fruit for each of her three children's school lunches. How many pieces of fruit did she buy for all of the school lunches each week?*

2. Following this, students were asked to form into groups of three to compare their categories and to identify and reflect upon the criteria they used to classify the problems.
3. The third activity required each group to join with another group forming groups of six students. Working collaboratively these larger groups required students to compare their categorisation criteria and to generate a similar word problem to illustrate each category.
4. Finally students were required to present their findings to the whole class and be prepared to justify their categorisation system.

In all, four interrelated tasks were given, viz

1. To classify the 36 given word problems into twelve different categories.
2. To generate criteria by which word problems may be categorised.

3. To generate a word problem reflecting these criteria for each of the twelve categories. To justify, in their own word, their choice of categorisation system and to explain how their own problems validated these criteria.

Results

Due to space constraints, the data for the six-student groups are only presented. Individual and three-student group responses parallel these findings.

Task 1: Categorisation. The initial task required students to categorise the 36 word problems into 12 different additive or multiplicative categories according to their own criteria. Table 2 shows the number of students who, working collaboratively, were able to correctly categorise the 36 word problems into categories similar to those shown in Table 1.

Table 2. Number of categories successfully classified by students.

Number of categories successfully categorised	Number of students n = 42
12	12
11	0
10	6
9	0
8	6
7	0
6	0
5	0
4	6
3	0
2	0
1	0
0	12

Table 2 indicates that even though 24 students (57%) were able to successfully classify 24 or more word problems, fewer than one-third of the students (29%) were able to correctly classify all word problems into twelve categories. Six students (14%) were able to correctly classify only 12 word problems and another six (14%) were unable to successfully classify any problems into categories. Anecdotal evidence suggests that these two groups of students lacked sufficient motivation to constructively participate in the activities.

Task 2: Criteria. This task involved students developing criteria by which the twelve groups of word problems may be categorised. Criteria used as a standard by which to evaluate student responses were based on Carpenter & Moser (1984) for additive word problems and Maguire (1996) for multiplicative word problems. These criteria arise from a semantic analysis of word problems. A full description of the semantic analysis of word problems is presented in Carpenter, et al.. (1984) and Maguire, (1996).
The criteria generated by students were classified as:

- Operation only - using only the arithmetic operation in order to classify the word problems. This would generate only four categories.
- Operation with additional features - using the arithmetical operation together with additional terms to differentiate between different categorisations involving the same operation.
- No criteria detailed - where the groups detailed no overt criteria by which the word problems could be categorised.

- Overtly stated criteria similar to those used as standard semantic features as outlined by Carpenter & Moser (1984) and Maguire (1996).

Table 3 illustrates the criteria developed by the students to classify the word problems into the twelve categories.

Table 3. Type of criteria generated and the number of students using these criteria.

Criteria	Number of students
Operation only	0
Operation with additional criteria.	30
Standard criteria	0
No criteria detailed	12

This table shows that no students relied solely on the arithmetic operation. The majority of students (30 students) used the arithmetic operation together with additional criteria to differentiate between the word problems which may be solved using the same operation. Twelve students detailed no criteria by which they classified the word problems into the twelve categories. No students were able to state criteria according to the standard semantic features of the word problems.

Intuitive criteria involving both an arithmetic operation and additional features to differentiate between problems requiring the same operation included such criteria as:

- addition in stages - first add one quantity and then following quantities;
- addition of identical objects - addition of "ducks" and "ducklings";
- addition of different objects - addition of "boys" and "girls";
- subtracting different things - subtraction of "boys" from "children";
- multiple groups of containers - seven boxes each containing three balls;
- dividing between people - number for each person.

Some students were able to categorise Cartesian Product problems as probability whilst others identified some word problems as missing addend.

Task 3: Word Problems: Students were required to generate a word problem which reflected the criteria they developed to classify the 12 different categories of word problems and to report their findings to the class.
Table 4 illustrates samples of student generated word problems.

Table 4. Student generated categories and an example of student generated word problem for these categories.

Category (as described by students)	Example
Add like objects	*Mary has 5 tazos. She won another 10. How many tazos does Mary now have?*
Adding different objects	*Lonnie has 5 rings and 2 necklaces. How many pieces of jewellery is Lonnie wearing?*
Subtraction to find difference	*Sena has 8 cocktails and Aranda has only 2. How many more cocktails does Aranda need to have to drink as much as Sena?*
Subtraction - how many (different items) more	*Irena has 100 guests for her wedding on the bride's side. There is 34 on the groom's side. How many more guests are on the bride's side?*
Comparative subtraction	*Jane has 6 boyfriends. Helen has only one boyfriend. How many boyfriends does Jane have to dump to have the same amount of boyfriends as Helen?*

Subtraction to find remainder	*Susanna had $50. She spent $35 on a Sportsgirl top. How much money does she have left?*
Probability	*Paula has 4 tiaras, 3 diamond rings and 2 ball gowns. How many outfits can she make altogether?*
Multiplication of lots	*Julia has 10 boxes of Lindor Balls and there are 6 balls in each box. How many Lindor Balls does she have?*
Comparative multiplication	*Jena has completed 2 of her assignments and Suzie has completed twice as many. How many assignments has Suzie completed?*
Division to find a number of people	*Jackie is having a party. She has 100 buritos. Jackie doesn't want any. Each guest gets 2 buritos. How many guests at the party?*
Division to allocate a number of items	*Judith is dating 10 Carlton footballers. She has 20 chocolate bars. If they are distributed equally how many bars does each footballer have?*
Comparative division	*Ira has 12 husbands. This is 3 times as many as Julia. How many husbands does Julia have?*

As can be seen from the above table this group of students was able to generate word problems for each of the 12 categories. Although the terminology differs from that of Table 1, they were able to create categories very similar to those shown in Table 1 and generate an example to illustrate each of their categories. Although the issue of gender within the social context in which word problems were generated was not a factor considered in this study, the very striking references to specific gender roles is worthy of further consideration in a later study. The group which generated these word problems was an all female group. Their word problems appear to reinforce a particular view of gender which may be considered counterproductive to acceptable pedagogical practice.
Task 4: Reflection and discussion. When a spokesperson for each group explained the group criteria and justified their choice to the class, much of the discussion revolved around the diversity of word problems presented. All students were surprised to discover that there were so many different categories of word problems. They were more surprised to learn that the sample was by no means exhaustive.

Discussion

It is apparent that although most students were unable to classify all the presented word problems into appropriate categories (30), those students (12) who were able to successfully cope with the demands of the first task were able to use the categories to devise intuitive criteria. These students were able to follow up their classification with the generation of suitable word problems using their intuitive criteria. It is of concern that no students were able to explicitly state their criteria in terms of a semantic analysis of the word problems. In stead students tended to rely upon the operations they saw as leading to a solution to the word problems. Where necessary students looked for additional features to further refine their criteria. These features (such as "more", "less") were extracted from the context of the problems.

It is quite obvious from Table 1 that all of the word problems presented to the students were those which could be solved using a straight algorithm without having to consider a realistic context within which the problems were situated. Had the word problems provided for these activities included more "realistic" or *problematic* contexts the students would have been introduced to problems more in tune with those likely to be met outside the classroom. This point will be further explored below. The results are in line with those found by Verschaffel et al. (1996).

The present study allowed pre-service teachers to consider word problems as a normal segment of the mathematics curriculum. As stated above, the pre-service teachers taking part in this study were not aware of the diversity of word problems available for inclusion in implementing their curriculum. This is in spite of the fact that word problems are referred to in the well publicised curriculum which they will be required to teach following graduation (Board of Studies, 1995). It is a curriculum outlined in the statements provided for teacher guidance by the Department of Educational, Victoria (Board of Studies, 1995).

Educational implications

There are two main implications of these results, viz

1. The continuing need to acquaint pre-service teachers with the role of word problems within the implemented curriculum.
2. The need to adopt what Verschaffel et al. (1996) refer to as "problematic" word problems so that students can learn to discover the mathematics of realistic situations and not regard school mathematics as something divorced from reality.

The role of word problems within the implemented curriculum

The Curriculum and Standards Framework: Mathematics (Board of Studies, 1995) (CSF) has detailed learning outcomes for students at each level of schooling. Within the CSF the following are instances of word problem learning outcomes identified:

- generate word problems using specified numbers and operations (CSF, p.54);
- generate further problems from familiar mathematical situations (CSF, p.104);
- ask questions to clarify the essential nature of a problem (CSF, p.104);
- identify key information in a problem and represent it using models, diagrams and lists (CSF, p.104).

Although the intended curricula of many countries state explicitly that problems solving in general and word problems in particular have an important part to play in the development of mathematical competence, anecdotal evidence gathered from my own teaching experiences suggests that neither teachers nor their students like generating word problems. Indeed teachers find it rather difficult to generate word problems (Maguire, 1996a), particularly within the primary school system, and many students are so unused to generating their own word problems that they have great difficulty in so doing (Maguire, 1998). In order to overcome these difficulties not only should mathematics methods courses include greater emphasis upon problem solving and generation but continual professional development resources should be devoted to training teachers in the use of problem solving within the classroom. This would allow both pre-service and their practising colleagues to become problem solvers themselves and, hopefully, allow them to benefit from research into problem solving already available.

Problematic word problems

Verschaffel et al. (1996) draws attention to an often neglected aspect of word problems i.e. a lack of real contexts within which text - and teacher - generated word problems are presented to students. An analysis of teacher generated word problems (Maguire, 1996a) indicates that not only had this author (as a class teacher) ignored many of the categories of word problems, and reinforced many of the misconceptions of mathematical operations to students (Greer, 1992), but also almost entirely neglected to include "problematic" word problems within the 200 word problems generated for this author's primary school class. This situation has the effect of divorcing school mathematics from real mathematics. Once again the amelioration of this unsatisfactory situation lies in the introduction of more "realism" into word problems provided to both pre-service and practising teachers.

One can posit the question "Is it any wonder that pre-service teachers are unable to elucidate classification criteria in order to categorise word problems?". The answer to this is "No!", particularly if they are not taught to see the mathematics of everyday situations within the classroom. It is evident to this researcher that school mathematics is not seen as real mathematics by many teachers or their students. The role of mathematics as a community skill is threatened by the generally poor results gained by many English speaking communities as shown in the recently released reports of the Third International Mathematics and Science Study (TIMSS) (Beaton et al., 1996). Even though Victorian primary school children scored higher than their cohort in the U.S., England and New Zealand, they were a long way behind the "leaders". The TIMSS report shows that

Australian children are falling behind their Asian neighbours in their mathematical competences. In order to overcome this lag in achievement additional research is required to determine whether knowledge of word problem categorisation (Carpenter & Moser, 1984; English, in press; Greer, 1992) and practice in problem generation and solving facilitates the understanding and incorporation of word problems into the repertoire of beginning teachers.

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